

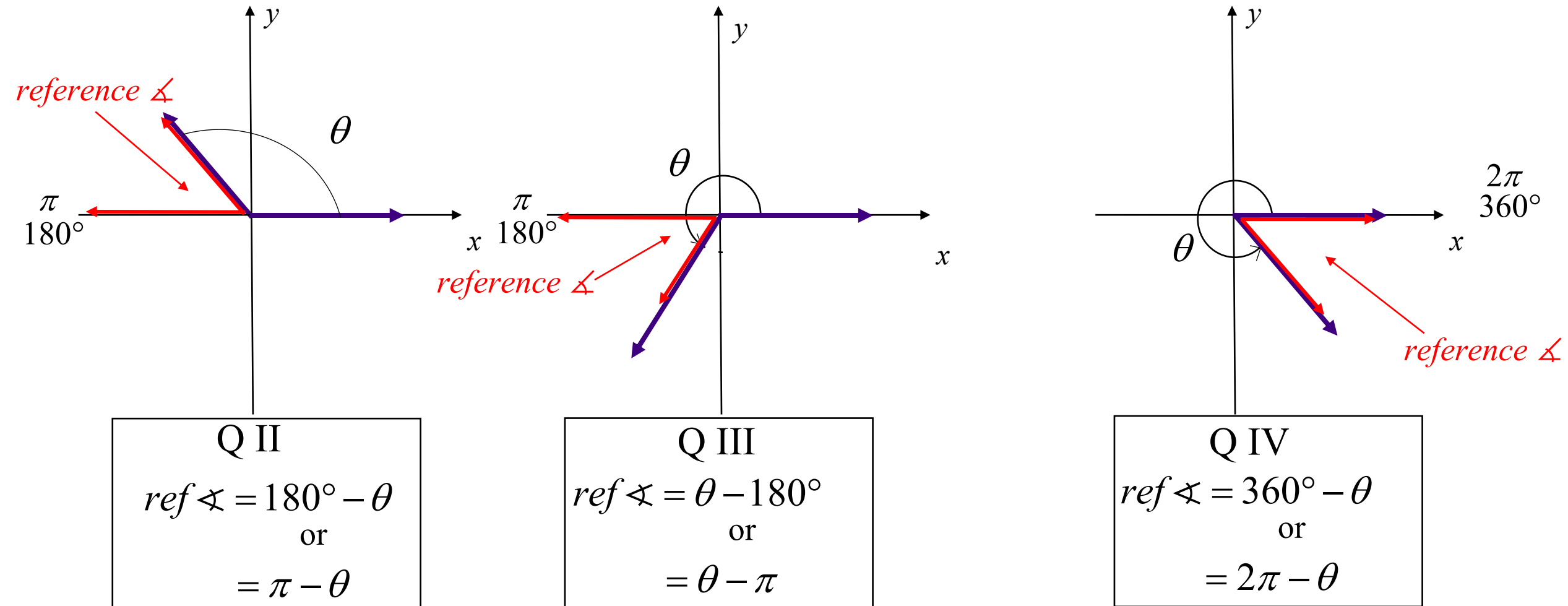
MAT123

Evaluating Trigonometric Functions

Finding Reference Angles

Reference Angles

When θ is a non-acute angle in standard position, its **reference angle** is the positive, acute angle formed by θ 's terminal side and the x-axis.

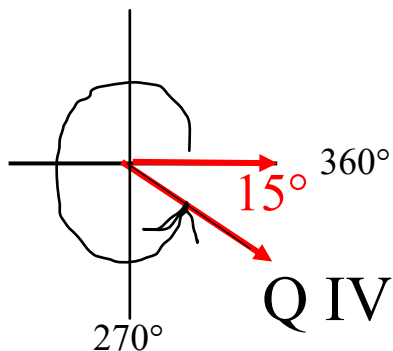


Finding Reference Angles

Find the reference angles of the following:

first make a quick sketch to determine quadrant

$$\theta = 345^\circ$$



$$\text{ref } \angle = 360^\circ - \theta$$

$$360 - 345 = 15^\circ \text{ reference } \angle$$

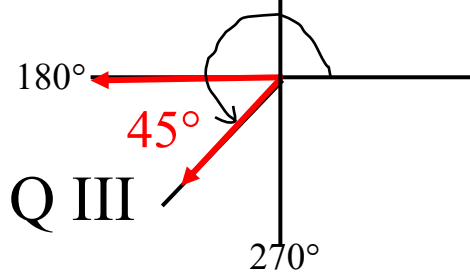
$$\theta = \frac{5\pi}{6} \quad (150^\circ)$$



$$\text{ref } \angle = \pi - \theta$$

$$\pi - \frac{5\pi}{6} = \frac{6\pi}{6} - \frac{5\pi}{6} = \frac{\pi}{6} \text{ reference } \angle \quad (30^\circ)$$

$$\theta = -135^\circ$$



$$\text{ref } \angle = \theta - 180^\circ$$

*first, find positive coterminal angle $-135 + 360 = 225^\circ$

$$225^\circ - 180^\circ = 45^\circ \text{ reference } \angle$$

Finding Reference Angles - Do

Find the reference angles of the following:

$$\theta = 210^\circ$$

$$= \boxed{30^\circ \text{ reference } \angle}$$

$$\theta = \frac{7\pi}{4}$$

$$= \boxed{\frac{\pi}{4} \text{ reference } \angle}$$

$$\theta = -240^\circ$$

$$= \boxed{60^\circ \text{ reference } \angle}$$

Reference Angles – Another Example

Find the reference angle of $\theta = \frac{17\pi}{3} - \frac{6\pi}{3} = \frac{11\pi}{3} - \frac{6\pi}{3} = \frac{5\pi}{3}$
still $> 2\pi$

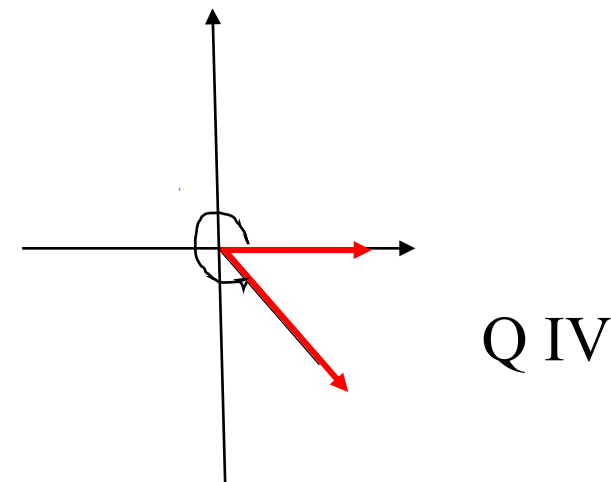
larger than one full rotation

$$2\pi = \frac{6\pi}{3} < \frac{17\pi}{3}$$

$$\text{ref } \angle = 2\pi - \theta$$

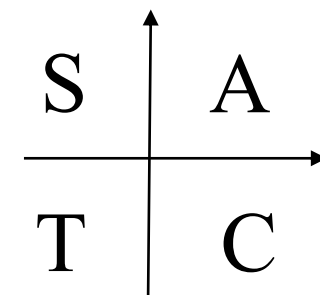
$$\begin{aligned} &= 2\pi - \frac{5\pi}{3} \\ &= \frac{6\pi}{3} - \frac{5\pi}{3} \end{aligned}$$

$$\text{reference } \angle = \frac{\pi}{3}$$



Another Example - Do

Do: Given $\tan \theta = -\frac{1}{5}$ and $\cos \theta < 0$, find $\sin \theta$ and $\sec \theta$. (Not on unit circle.)



Evaluate Using Reference Angles

Step 1: sketch angle to determine quadrant

Step 2: determine sign from quadrant

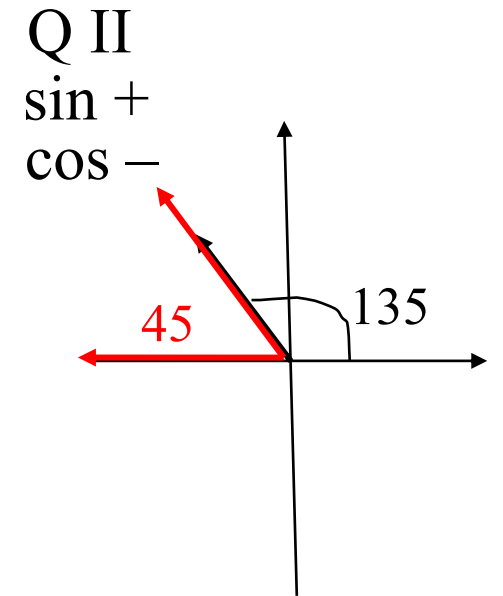
Step 3: determine reference angle

Step 4: re-write function

Step 5: evaluate function

ex. evaluate $\sin 135^\circ = +\sin 45^\circ = \boxed{\frac{\sqrt{2}}{2}}$ ex. evaluate $\cos 135^\circ = -\cos 45^\circ = \boxed{-\frac{\sqrt{2}}{2}}$

look up
on chart



ex. evaluate $\cos \frac{4\pi}{3} = -\cos \frac{\pi}{3} = \boxed{-\frac{1}{2}}$ Q III
cos -

$\frac{\pi}{3}$

ex. evaluate $\cot\left(-\frac{\pi}{3}\right) = -\cot \frac{\pi}{3} = -\frac{\cos \frac{\pi}{3}}{\sin \frac{\pi}{3}} = -\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} = -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3} = \boxed{-\frac{\sqrt{3}}{3}}$ Q IV
cot -

Evaluate Using Reference Angles - Do

Step 1: sketch angle to determine quadrant

Step 2: determine sign from quadrant

Step 3: determine reference angle

Step 4: re-write function

Step 5: evaluate function

Do: evaluate $\cos 150^\circ$

Do: evaluate $\sin \frac{11\pi}{6}$

Do: evaluate $\sec(405^\circ)$